



Q.1. Solve the following questions.

(15x2=30)

- (2 points) Let $f(x) = \frac{\sqrt{x^2-16}}{x+12}$, $x \neq -12$. Find domain and range of f .
- (2 points) Given $f(x) = x^3 + ax^2 + bx + 1$. If $f(2) = -3$ and $f(-1) = 0$. Find the value of a and b .
- (2 points) Find inverse function of $f(x) = 2 + \sqrt{x-1}$.
- (2 points) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 7x}{x}$.
- (2 points) Draw graph of $y = \begin{cases} x+3 & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$
- (2 points) If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.
- (2 points) Show that $\begin{vmatrix} x & a+x & b+c \\ x & b+x & c+a \\ x & c+x & a+b \end{vmatrix} = 0$.
- (2 points) Show that, $A = \begin{bmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{bmatrix}$ is a singular matrix.
- (2 points) What is the circular measure of the angle between the hands of a watch at 4 O'clock?
- (2 points) Prove that $\cot^4 \theta + \cot^2 \theta = \csc^4 \theta - \csc^2 \theta$, where θ is not an integral multiple of $\frac{\pi}{2}$.
- (2 points) Find the value of $\cos \frac{\pi}{12}$.
- (2 points) Prove that $\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$.
- (2 points) Find the period of $3 \cos \frac{\pi}{5}$.
- (2 points) Find the center and radius of the circle having equation,
$$4x^2 + 4y^2 - 8x + 12y - 25 = 0.$$
- (2 points) Find the center, foci, eccentricity, vertices and equations of the directrices of the ellipse having equation, $9x^2 + y^2 = 18$.

Q.2. Solve the following questions.

(5x6=30)

1. Prove that, $\lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^n = e$.

2. Show that
$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^2.$$

3. Find rank of the matrix, $A = \begin{bmatrix} 3 & 2 & -9 & 5 \\ 4 & 8 & 9 & -1 \\ 3 & -1 & 0 & -1 \\ 0 & 3 & 0 & 0 \end{bmatrix}$

4. Prove that $(\tan \theta + \cot \theta)^2 = \sec^2 \theta \csc^2 \theta$.

5. Prove that $\frac{1 - \tan \theta \tan \phi}{1 + \tan \theta \tan \phi} = \frac{\cos(\theta + \phi)}{\cos(\theta - \phi)}$.