UNIVERSITY OF THE PUNJAB

Roll No. ....

Sixth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Linear Algebra (MA) **TIME ALLOWED: 30 mins.**
Course Code: IT-312 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1 **MCQs (1x10 = 10 Marks)**

(i) The set $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ of vectors in R^2 is ----------

a) Linearly Independent b) Linearly dependent c) Basis of R^2 d) None of these

(ii) A system $Ax = b$ of m linear equations in n unknowns in n variables has a solution if and only if----------

(a) $rank\ of\ A = rank\ of\ A_b$ (b) $rank\ of\ A \neq rank\ of\ A_b$

(c) $rank\ of\ A = m$ (d) None of these

(iii) If A is a matrix of order 3×3 and $\det(A) = 3$, then the value of $\det(2A)$ is --------

(a) 24 (b) 27 (c) 54 (d) None of these

(iv) A unit vector orthogonal to both $(1, 1, 2)$ and $(0, 1, 3)$ in R^3 is ----------

(a) $\left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ (b) $\left(\frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ (c) $\left(\frac{2}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}\right)$ (d) $\left(\frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$

(v) The set $W = \{(x, y, z) \in R^3 : x + y + z = c\}$ is a subspace of R^3 if ----------

(a) $c > 0$ (b) $c < 0$ (c) $c = 0$ (d) None of these

(vi) The property $\forall$ a, b$\in$R then $a + b = b + a$ is called ----------

(a) Associative property (b) Transitive property

(c) Closure property (d) None of these

(vii) A linear transformation $T : U \to V$ is one-to-one if and only if ---------

(a) $N(T) = \{0\}$ (b) $N(T) \neq \{0\}$ (c) $N(T) = \{1\}$ (d) $N(T) = \{-1\}$

(viii) The transformation $T : R^3 \to R^2$ be defined by $T(x_1, x_2, x_3) = (x_1 + 1, x_2 + x_3)$ is

(a) Linear (b) Not Linear (c) Rational (d) None of these

(ix) The dimension of $KerT$ is called ----------

(a) Rank (b)Nullity (c) basis (d)} none of these

(x) The characteristic polynomial of the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ is----------

(a) $p(\lambda) = (3 - \lambda)^2$ (b) $p(\lambda) = (3 - \lambda)(4 - \lambda)$

(c) $p(\lambda) = \lambda^2$ (d) None of these

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2017
Examination: B.S. 4 Years Programme Roll No. ....

PAPER: Linear Algebra (MA) **TIME ALLOWED: 2 hrs. & 30 mins.**
Course Code: IT-312 **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2 **SHORT QUESTIONS** **(4x5 = 20 Marks)**

(i) Solve the system of linear equations

$$6x-6y+6z=6$$
$$2x-4y-6z=12$$
$$10x-5y+5z=30$$

(ii) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{c} & a \\ b & b & \frac{c^2+a^2}{c} \end{vmatrix} = 4abc$

(iii) Show that the vectors $(1,2)$, and $(3,5)$ span the vector space $\mathbb{R}^2$

(iv) Define $T: R^3 \rightarrow R^3$ by $T(x_1,x_2,x_3)=(x_1-x_2, x_1+x_3, x_2+x_3)$. Find the basis and dimension of $N(T)$.

(v) Find inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

SECTION-III

LONG QUESTIONS **(6x5 = 30 Marks)**

Q.3 Solve the system of linear equations by Gaussian elimination method

$x+y+2z=9, \quad 2x+4y-3z=1, \quad 3x+6y-5z=0$

Q.4 Find the characteristic polynomial, eigen Values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

Q.5 Show that the matrix $\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent.

Q.6 Determine whether or not the given set of vectors is a basis of R^3
$v_1=(1,2,-1), v_2=(0,3,1), v_3=(1,-5,3)$.

Q.7 Write $v=(1,-2,5)$ as a linear combination of the vectors
$v_1=(1,1,1), v_2=(1,2,3), v_3=(2,-1,1)$

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Linear Algebra (MA) **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: IT-312 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2 **SHORT QUESTIONS** **(4x5 = 20 Marks)**

(i) Determine the values of a for which the system of linear equations has no solution, exactly one solution and infinitely many solutions.

$x + y + 7z = -7$

$2x + 3y + 17z = -16$

$x + 2y + (a^2 + 1)z = 3a.$

(ii) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{c} & a \\ b & b & \frac{c^2+a^2}{c} \end{vmatrix} = 4abc$

(iii) Show that the vectors $(1,-2)$, and $(3,-5)$ span the vector space $\mathbb{R}^2$

(iv) Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one?

(v) Show that the matrix $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutery

SECTION-III

LONG QUESTIONS **(6x5 = 30 Marks)**

Q.3 If A and B are 3×3 matrices such that $\det(A^2B^3) = 108$ and $\det(A^3B^2) = 72$ then find $\det(2A)$ and $\det(B^{-1})$.

Q.4 Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal, where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q.5 If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

Q.6 Show that any finite dimensional vector space contains a basis.

Q.7 Determine whether or not the given set of vectors is a basis of $\mathbb{R}^3$

$\{(1,2,-1),(0,3,1),(1,-5,3)\}$

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Linear Algebra (MA) **TIME ALLOWED: 15 Mints.**

Course Code: IT-312 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. 1 **MCQs (1x10 = 10 Marks)**

(i) The set $S=\{(1,2),(2,3),(0,0)\}$ of vectors in R^2 is ----------

(a) linearly independent (b) linearly dependent (c) basis of R^2 (d) None of these

(ii) The basis of trivial subspace $A=\{0\}$ is ----------

(a) $\{0\}$ (b) $\{1\}$ (c) $\{\ \}$ (d) None of these

(iii) If A is a matrix of order 3×3 and $\det(A)=-2$, then the value of $\det(3A)$ is --------

(a) -24 (b) -6 (c) -27 (d) -54

(iv) A system of m homogeneous linear equations $Ax=0$ in n variables has a non-trivial solution if and only if the rank of A is ----------

(a) equal to n (b) less than n (c) greater to n (d) None of these

(v) The subspace of R^3 spanned by the vector (a, b, c) is ----------

(a) $x=t, y=bt, z=ct$ (b) $x=-at, y=-bt, z=-ct$

(c) $x=at, y=bt, z=ct$ (d) None of these

(vi) The property $\forall$ a, b$\in$R then $a+b\in R$ is called

(a) Associative property (b) Transitive property

(c) Closure property (d) None of these

(vii) A linear transformation $T:U\to V$ is one-to-one if and only if ----------

(a) $N(T)=\{0\}$ (b) $N(T)\neq\{0\}$ (c) $N(T)=\{1\}$ (d) $N(T)=\{-1\}$

(viii) Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T: R^3\to R^3$ defined by $T(x,y,z)=(x,y,0)$ is

a) Linear b) Not Linear c) Rational d) None of these

(ix) The dimension of $KerT$ is called ....

(a) Rank (b)Nullity (c) basis (d)} none of these

(x) The characteristic polynomial of the matrix $\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$ is...............

(a) $p(\lambda)=(1-\lambda)^2$ (b) $p(\lambda)=(2-\lambda)(3-\lambda)$

(c) $p(\lambda)=\lambda^2$ (d) None of these

...E PUNJAB

Sixth Semester 2014

Examination: B.S. 4 Years Programme

Roll No. 005075

PAPER: Linear Algebra (MA)

Course Code: IT-312

TIME ALLOWED: 2 hrs. & 30 mins.

MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

solve the short answer 10x2=20

1) find the trace of the matrix

$$A = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$$

2) Determine test for invertibility

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & 4 & 6 \end{bmatrix}$$

3) Find the norm of $v=(3,3,1)$
4) Define characteristic equation of A
5) If $T: V \to W$ is a linear transformation then prove $T(u-v) = T(u) - T(v)$ for all u and v in V
6) Write the standerd matrix for transformation $T(x_1, x_2) = (-x_1+x_2, x_1-x_2)$
7) Let $u=(-3,2,1,0)$, $v=(4,7,-3,2)$ and $w=(5,-2,8,1)$ find $-u+(v-4w)$
8) Define solution spaces of homogeneous system
9) $T_1: U \to V$, $T_2: V \to W$, $T_3: W \to Y$ are linear transformation, then define $T_3 o T_2 o T_1$
10) Without direct evaluation show that

$$\det \begin{bmatrix} b+c & c+a & b+a \\ a & b & c \\ 1 & 1 & 1 \end{bmatrix} = 0$$

Slove all the question. All question carry equal marks 6x5 =30

1) What conditions must b_1, b_2 and b_3 satisfy in order for the system of equation $x_1 + x_2 + 2x_3 = b_1$, $x_1 + x_3 = b_2$, $2x_1 + x_2 + 3x_3 = b_3$ to be consistent.
2) Let $u=(2,-1,3)$ and $a=(4,-1,2)$ find the vector component of u along a and the vector component of u orthogonal to a.
3) Solve

$$\begin{aligned} x+y+2z&=9 \\ 2x+4y-3z&=1 \\ 3x+6y-5z&=0 \end{aligned}$$

By Gaussian elimination and back substitution.

4) Determine whether $v_1=(1,1,2)$, $v_2=(1,0,1)$, $v_3=(2,1,3)$ span the vector space R^3
5) Find a matrix p that diagonalizes $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$
6) Let $T: R^3 \to R^3$ be the linear operator defined by the formula $T(x_1, x_2, x_3) = (3x_1+x_2, -2x_1-4x_2+3x_3, 5x_1+4x_2-2x_3)$ determine whether T is one to one if so find $T^{-1}(x_1, x_2, x_3)$

UNIVERSITY OF THE PUNJAB

Sixth Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ...7823.......

PAPER: Linear Algebra (MA)
Course Code: IT-312

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2 **SHORT QUESTIONS** **(4x5 = 20 Marks)**

(i) Find the reduced echelon form of the matrix

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

(ii) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

(iii) Show that the vectors $(1-i, i)$, and $(2, -1+i)$ C^2 are linearly independent over C but linearly independent over R.

(iv) Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one?

(v) Show that the matrix ~~[matrix]~~ ~~is involutory~~ $\begin{bmatrix} 0 & 1 & 1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutory.

SECTION-III

LONG QUESTIONS **(6x5 = 30 Marks)**

Q.3 Solve the system of linear equations

$2x + y + z = 1$, $3x + y - 5z = 8$, $4x - y + z = 5$

Q.4 Show that $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2y^2$

Q.5 If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ determine the value of $A^2 - 4A - 5I$

Q.6 Determine whether or not the set of vectors $\{(1,2,-1), (0,3,1), (1,-5,3)\}$ is a basis for R^3?

Q.7 Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

UNIVERSITY OF THE PUNJAB

Sixth Semester 2016
Examination: B.S. 4 Years Programme

Roll No. 00943

PAPER: Linear Algebra (MA)
Course Code: IT-312

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2 SHORT QUESTIONS (4x5 = 20 Marks)

(i) Solve the following system of linear equations by using matrix inversion method

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 + 5x_2 + 3x_3 &= 3 \\ x_1 \qquad + 8x_3 &= 17 \end{aligned}$$

(ii) Use Cramer's rule to solve the following system of linear equations

$$\begin{aligned} x_1 + \qquad + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned}$$

(iii) Discuss whether the vectors $(1,2)$, and $(3,-5)$ span the vector space $\mathbb{R}^2$?

(iv) Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one?

(v) Show that the matrix $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutery

SECTION-III

LONG QUESTIONS (6x5 = 30 Marks)

Q.3 Solve the system of linear equations by Gaussian elimination method

$x + y + 2z = 9, \quad 2x + 4y - 3z = 1, \quad 3x + 6y - 5z = 0$

Q.4 Find the Eigen Values and eigen vectors of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

Q.5 If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

Q.6 Determine whether the vectors are linearly independent or not?
$v_1 = (1,-2,3), v_2 = (5,6,-1), v_3 = (3,2,1)$.

Q.7 Write $v = (1,-2,5)$ as a linear combination of the vectors
$v_1 = (1,1,1), v_2 = (1,2,3), v_3 = (2,-1,1)$

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. 014694........

PAPER: Linear Algebra (MA)

Course Code: IT-312 Part – II

TIME ALLOWED: 2 Hrs. & 45 Mints.

MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Q. 2 SHORT QUESTIONS (4x5 = 20 Marks)

(i) Determine the values of a for which the system of linear equations has no solution, exactly one solution and infinitely many solutions.

$x+y+7z=-7$

$2x+3y+17z=-16$

$x+2y+(a^2+1)z=3a.$

(ii) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{c} & a \\ b & b & \frac{c^2+a^2}{c} \end{vmatrix} = 4abc$

(iii) Show that the vectors $(1,-2)$, and $(3,-5)$ span the vector space $\mathbb{R}^2$

(iv) Define $T: R^3 \to R^3$ by $T(x_1,x_2,x_3)=(-x_3,x_1,x_1+x_3)$. Find $N(T)$. Is T one-to-one?

(v) Show that the matrix $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutery

SECTION-III

LONG QUESTIONS (6x5 = 30 Marks)

Q.3 If A and B are 3×3 matrices such that $\det(A^2B^3)=108$ and $\det(A^3B^2)=72$ then find $\det(2A)$ and $\det(B^{-1})$.

Q.4 Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal, where $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q.5 If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

Q.6 Show that any finite dimensional vector space contains a basis.

Q.7 Determine whether or not the given set of vectors is a basis of $\mathbb{R}^3$

$\{(1,2,-1),(0,3,1),(1,-5,3)\}$

UNIVERSITY OF THE PUNJAB

Sixth Semester - 2017
Examination: B.S. 4 Years Programme

Roll No. 12012

PAPER: Linear Algebra (MA)
Course Code: IT-312

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2

SHORT QUESTIONS (4x5 = 20 Marks)

(i) Solve the system of linear equations

$6x-6y+6z=6$

$2x-4y-6z=12$

$10x-5y+5z=30$

(ii) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{c} & a \\ b & b & \frac{c^2+a^2}{c} \end{vmatrix} = 4abc$

(iii) Show that the vectors $(1,2)$, and $(3,5)$ span the vector space $\mathbb{R}^2$

(iv) Define $T: R^3 \to R^3$ by $T(x_1,x_2,x_3)=(x_1-x_2, x_1+x_3, x_2+x_3)$. Find the basis and dimension of $N(T)$.

(v) Find inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

SECTION-III

LONG QUESTIONS (6x5 = 30 Marks)

Q.3 Solve the system of linear equations by Gaussian elimination method

$x+y+2z=9, \quad 2x+4y-3z=1, \quad 3x+6y-5z=0$

Q.4 Find the characteristic polynomial, eigen Values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

Q.5 Show that the matrix $\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent.

Q.6 Determine whether or not the given set of vectors is a basis of R^3

$v_1=(1,2,-1), v_2=(0,3,1), v_3=(1,-5,3)$.

Q.7 Write $v=(1,-2,5)$ as a linear combination of the vectors

$v_1=(1,1,1), v_2=(1,2,3), v_3=(2,-1,1)$

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – Spring 2022

Roll N

Course Code: IT-312

Paper: Linear Algebra (MA)

Time: 3 Hrs. Marks: 60

THE ANSWERS MUST BE ATTEMPTED ON THE ANSWER SHEET PROVIDED

Q.1. *Answer the following short questions.* (6x5=30)

(*i*) Define reduced row echelon form and give 3 examples.

(*ii*) Find a row operation and the corresponding elementary matrix that will restore the elementary matrix $\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$ to the identity matrix.

(*iii*) Confirm the identity without evaluating the determinant directly.

$$\begin{vmatrix} a_1 + b_1 t & a_2 + b_2 t & a_3 + b_3 t \\ a_1 t + b_1 & a_2 t + b_2 & a_3 t + b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1 - t^2) \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(*iv*) Determine whether the polynomials $2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$ in P_2 are linearly dependent.

(*v*) Discuss how the rank of $A = \begin{bmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{bmatrix}$ varies with t.

(*vi*) **The eigenvalues of a matrix A are the same as the eigenvalues of the reduced row echelon form of A. Determine wheter the statement is true or false, and justify your answer.**

Answer the following questions. (3x10=30)

Q.2. Solve the following system of nonlinear equations for x, y, and z.

$$\begin{aligned} x^2 + y^2 + z^2 &= 6 \\ x^2 - y^2 + 2z^2 &= 2 \\ 2x^2 + y^2 - z^2 &= 3 \end{aligned}$$

Q.3. Find the coordinate vector of A relative to the basis $S = \{A_1, A_2, A_3, A_4\}$.

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}; A_1 = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Q.4. Find a matrix P that diagonalizes A, and compute $P^{-1}AP$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Paper: Linear Algebra (MA)
Course Code: IT-312 Part – I (Compulsory) **Time: 30 Min. Marks: 10**

Roll No. in Fig. ....
Roll No. in Words. ....
....
....
Signature of Supdt.:

ATTEMPT THIS PAPER ON THIS QUESTION SHEET ONLY.
Division of marks is given in front of each question.
This Paper will be collected back after expiry of time limit mentioned above.

Q.1. Encircle the correct choice. (1x10=10)

(i) If A and B are two square matrices of the same size then
(a) $tr(AB) = tr(A)tr(B)$ (b) $tr(AB) \neq tr(A)tr(B)$
(c) $tr(AB) = tr(A) + tr(B)$ (d) None of these

(ii) A system of m homogeneous linear equations $Ax = 0$ in n variables has a non-trivial solution if and only if the rank of A is ----------
(a) equal to n (b) less than n (c) greater to n (d) None of these

(iii) If A is a matrix of order 3×3 and $\det(A) = 2$, then the value of $\det(3A)$ is --------
(a) -24 (b) -6 (c) -27 (d) None of these

(iv) A group has exactly ---------- identity element.
(a) One (b) Two (c) Three (d) Four

(v) The subspace of R^3 spanned by the vector (a, b, c) is ----------
(a) $x = t, y = bt, z = ct$ (b) $x = -at, y = -bt, z = -ct$
(c) $x = at, y = bt, z = ct$ (d) None of these

(vi) The property $\forall$ a, b, c $\in$ R then $a + (b + c) = (a + b) + c$ is called
(a) Associative property (b) Transitive property
(c) Closure property (d) None of these

(vii) A linear transformation $T : U \to V$ is one-to-one if and only if ----------
(a) $N(T) = \{0\}$ (b) $N(T) \neq \{0\}$ (c) $N(T) = \{1\}$ (d) $N(T) = \{-1\}$

(viii) Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is
a) Linear b) Not Linear c) Rational d) None of these

(ix) The dimension of $KerT$ is called ....
(a) Rank (b)Nullity
(c) basis (d)} none of these

(x) The characteristic polynomial of the matrix $\begin{pmatrix} 4 & 0 \\ 0 & 7 \end{pmatrix}$ is................
(a) $p(\lambda) = (2 - \lambda)^2$ (b) $p(\lambda) = (4 - \lambda)(7 - \lambda)$
(c) $p(\lambda) = (4 + \lambda)(7 + \lambda)$ (d) None of these

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2019

Roll No. ....

Paper: Linear Algebra (MA)
Course Code: IT-312 Part – II

Time: 2 Hrs. 30 Min. Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q. 2 SHORT QUESTIONS (4x5 = 20 Marks)

(i) Find the reduced echelon form of the matrix

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

(ii) Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

(iii) Show that the vectors $(1-i, i)$, and $(2, -1+i)$ C^2 are linearly independent over C but linearly independent over R.

(iv) Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one?

(v) Find the Eigen Values and eigen vectors of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

LONG QUESTIONS (6x5 = 30 Marks)

Q.3 Solve the system of linear equations

$$3x + 2y + 4z = 7$$
$$2x + y + z = 4 \quad 3x+2y+4z=7$$
$$x + 3y + 5z = 3$$

Q.4 Show that $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2 y^2$

Q.5 If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ determine the value of $A^2 - 4A - 5I$

Q.6 Determine whether or not the set of vectors $\{(1,2,-1),(0,3,1),(1,-5,3)\}$ is a basis for R^3?

Q.7 Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program / Sixth Semester – 2020

Roll No. [redacted]

Paper: Linear Algebra (MA)
Course Code: IT-312 Part – II

Time: 2 Hrs. 30 Min. Marks: 50

ATTEMPT THIS (SUBJECTIVE) ON THE SEPARATE ANSWER SHEET PROVIDED

Q. 2 **SHORT QUESTIONS** **(4x5 = 20 Marks)**

(i) Solve the following system of linear equations by using Gauss Jordan elimination method

$2x_1 - x_2 - x_3 = 4,\ 3x_1 + 4x_2 - 2x_3 = 11,\ 3x_1 - 2x_2 + 4x_3 = 11$

(ii) Prove that the following determinant vanishes

$$\begin{vmatrix} bc & ca & ab \\ 1/a & 1/b & 1/c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

(iii) Discuss whether the vectors $(1,2)$, and $(3,-5)$ span the vector space R^2?

(iv) Define $T: R^3 \to R^3$ by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$. Find the basis and dimension of $R(T)$.

(v) Show that the matrix $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutery

SECTION-III

LONG QUESTIONS **(6x5 = 30 Marks)**

Q.3 Solve the system of linear equations by Gaussian elimination method

$x + y + 2z = 9,\qquad 2x + 4y - 3z = 1,\qquad 3x + 6y - 5z = 0$

Q.4 Find the Eigen Values and eigen vectors of $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$

Q.5 Reduce the matrix $\begin{bmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{bmatrix}$

Q.6 Determine whether the vectors are linearly independent or not?
$v_1 = (1,-2,4,1), v_2 = (2,1,0,-3), v_3 = (1,-6,1,4)$.

Q.7 Determine whether or not the given set of vectors is a basis of R^3
$v_1 = (1,2,-1), v_2 = (0,3,1), v_3 = (1,-5,3)$.

UNIVERSITY OF THE PUNJAB

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Q.2. Solve the following. (2x10=20)

1. (10 points) Given a $p \times q$ matrix A with rank r, what is the dimensions of column space of A? How can we find out the basis of the column space of any matrix? Explain with an example.
2. (10 points) Give an example of a 3-D space which is not R^3.

Q.3. Solve the following. (2x15=30)

1. (15 points) Find the projection matrix for the column space of a 4×2 matrix A given below:

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2. (15 points) How can we figure out that given a set of vectors, the set is an independent set or not? Explain with an example.