UNIVERSITY OF THE PUNJAB

Third Semester 2012

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT)

Course Code: IT-21404

TIME ALLOWED: 2 hrs. & 30 mins.

MAX. MARKS: 60

Attempt this Paper on Separate Answer Sheet provided.

Subjective Part

Question-2 (5+5+5+5+5+5)

a) **Write converse and contra positive statement** of "'two angles are a linear pair only if they are supplementary".

b) Check whether the given graph is W_7 complete bipartite or not.

c) Suppose a programming language requires you to define variable names (identifiers) using exactly 3 different upper case characters. How many different identifiers contain either an A or a B, but not both?

d) Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

e) Draw graph of the following adjacency matrix

$$\begin{bmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

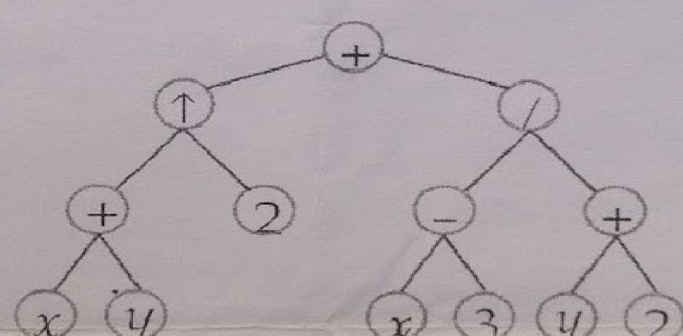
f) Show that $1^2 + 2^2 .... n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers.

Question-3 (10+10+10)

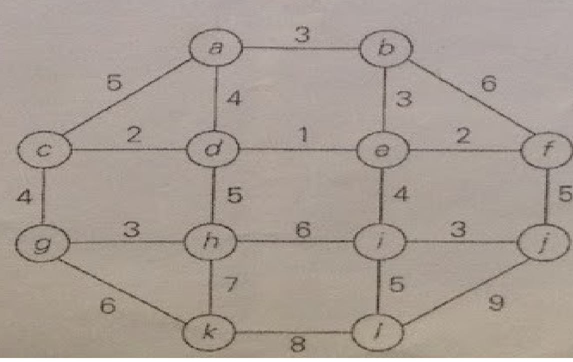
a) **Consider P(x) :"x can speak Russian" and let Q(x): x knows c++" Convert the following statements into symbolic form by using Quantifiers and predicates. First take universe of discourse ={students in your school" then take U.D ={all people}**

a. There is a student in your school who can speak Russian and who knows C++.
b. No student at your school either can speak Russian and knows C++.
c. Not all students at your school can speak Russian.
d. There is a student in your school who can speak Russian but do not know C++.
e. Every student at your school knows C++.

b) List the Pre-order and in order , post order traversal and in order traversal of the following tree.



c) Apply Prim's and kruskal's algorithm to find a spanning tree from the following graph.



UNIVERSITY OF THE PUNJAB

Third Semester 2013
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT)
Course Code: IT-21404 / MATH-231

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Short Questions

Q2 (10×2)

(i) Show that there are no solutions in integers x and y of $x^2 + 3y^2 = 8$.

(ii) Let $f : R \to R$ be defined by $f(x) = \frac{5x-7}{3}$. Find a formula for f^{-1}.

(iii) Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

(iv) Define one-to-one and onto functions.

(v) Determine whether the function $f(x) = x^3$ form $R \to R$ is one-to-one? Is this function onto?

(vi) What is the Cartesian product $A \times B \times C$, where $A = \{0,1\}$, $B = \{1,2\}$, and $C = \{0,1,2\}$?

(vii) Find the prime factorization of 45617.

(viii) List five integers that are congruent to 4 modulo 12.

(ix) If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, what is their least common multiple?

(x) Find the solution of the linear homogeneous recurrence relation $a_n = 7a_{n-1} - 6a_{n-2}$ with $a_0 = -1$ and $a_1 = 4$.

Long Questions

Q3 (10)

Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.

Define a graph and a tree. Also draw the graphs of $K_{3,3}$ and W_6.

(10)

Show that the implication $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology by using truth table.

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg p$ are logically equivalent.

(10)

Let $R_1 = \{(1,1), (1,3), (2,2), (3,1)\}$, $R_2 = \{(1,1), (3,3), (2,2), (3,1)\}$, $R_3 = \{(1,2), (3,3), (2,1)\}$, $R_4 = \{(1,3), (2,3)\}$ be the relations on $\{1, 2, 3\}$. Then,

a) Which of these relations are reflexive? Justify your answer.

b) Which of these relations are symmetric? Justify your answer.

c) Which of these relations are antisymmetric? Justify your answer.

d) Which of these relations are transitive? Justify your answer.

UNIVERSITY OF THE PUNJAB

Third Semester 2014

Examination: B.S. 4 Years Programme

Roll No. 7312

PAPER: Discrete Mathematics (IT)
Course Code: MATH-231

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Short Questions

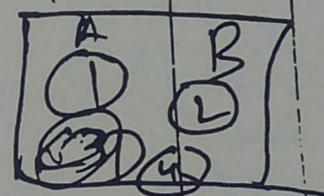
Q2 (10×2)

i) Let $f : R \to R$ be defined by $f(x) = \frac{5x-7}{3}$. Find a formula for f^{-1}.

ii) Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

iii) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

iv) Find all partitions of the set $S = \{1, 2, 3\}$ and show by Venn diagram.



v) Define one-to-one and onto functions.

vi) Determine whether the function $f(x) = x^2$ form the set of integers to the set of integers is one-to-one? Is this function onto?

vii) Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

viii) List five integers that are congruent to 4 modulo 12.

ix) Draw the graph of $K_{3,4}$ and W_7.

x) Let $A = \{0, 1, 2, 3\}$ and $R = \{(0,0), (1,1), (1,3), (2,2), (2,3), (3,1), (3,2), (3,3)\}$. Show that R is an equivalence relation.

Long Questions

Q3 (10)

a) Use mathematical induction to show $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n.

b) Define a graph and a tree. Also write vertices and edges of the graph K_n.

Q4 (10)

a) Show that the implication $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology by using truth table.

b) Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

Q5 (10)

a) Give a formula for the coefficients of x^k, k is an integer, in the expansion of $(x^2 - \frac{1}{x})^{100}$.

b) Draw the graph whose adjacency matrix is given by

$$\begin{pmatrix} 2 & 2 & 3 & 1 \\ 2 & 0 & 0 & 4 \\ 3 & 0 & 2 & 2 \\ 1 & 4 & 2 & 0 \end{pmatrix}$$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2015
Examination: B.S. 4 Years Programme

PAPER: Discrete Mathematics (IT)
Course Code: MATH-231

TIME ALLOWED: 30 mins.
MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

OBJECTIVE

Q1 Tick on the correct option 10

i) $\neg p \vee \neg q$ is logically equivalent to

a) $\neg p \wedge \neg q$ b) $\neg(p \wedge q)$ c) $\neg p \wedge q$ d) $p \wedge \neg q$

ii) Number of edges in K_4 are

a) 4 b) 5 c) 6 d) 8

iii) Number of strings can be made by reordering the letters of SUCCESS.

a) 340 b) 420 c) 512 d) 625

iv) How many permutations of letters ABCDE contain the string ABC.

a) 3! b) 4! c) 5! d) 6!

v) Graphs that a number assigned to each edge are called…………….graphs

a) complete b) weighted c) simple d) bipartite

vi) The cardinality of the set $A = \{a, \{a\}, \{a, \{a\}\}\}$ is

a) 3 b) 4 c) 2 d) 1

vii) The set $P(\{a, b, \{a, b\}\})$ has elements

a) 4 b) 8 c) 12 d) 16

viii) $\overline{A \cup (B \cap C)} =$

a) $(\overline{C} \cup \overline{B}) \cap \overline{A}$ b) $(\overline{A} \cup \overline{B}) \cap \overline{C}$ c) $(\overline{C} \cup \overline{A}) \cap \overline{B}$ d) $(\overline{C} \cap \overline{B}) \cup \overline{A}$

ix) The domain of the function $f(x) = \sqrt{|x|}$ is

A) $(-\infty, 0]$ b) $[0, \infty)$ c) $(-\infty, \infty)$ d) all of these

x) If both f and g are one-to-one functions, then $f \circ g$ is

a) one-to-one b) onto c) a & b d) none of these

UNIVERSITY OF THE PUNJAB

Third Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT)
Course Code: MATH-231

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE

Q2	i) Let $f: R^{+} \to R$ be defined by $f(x)=\frac{3\sqrt{x}-1}{2}$. Find a formula for f^{-1}. ii) Prove that an undirected graph has an even number of vertices of odd degree. iii) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent. iv) Let a, b and c be positive integers. Prove that if $a \mid b$ and $b \mid c$ then $a \mid c$. v) Define one-to-one and onto functions. vi) Let $f: R \to R$ be defined by $f(x)=x^2$. Determine whether $f(x)$ is one-to-one? Is this function onto? vii) Find the prime factorization of 45617. viii) List five integers that are congruent to 3 modulo 11. ix) Draw the graph of $K_{5,4}$ and W_7. x) Let $A=\{0,1,2,3\}$ and $R=\{(0,0),(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3)\}$. Show that R is an equivalence relation.	10×2
	Long Questions	
Q3	a) Use mathematical induction to show that $1^3+2^3+3^3+\cdots+n^3=\left(\frac{n(n+1)}{2}\right)^2$ for all positive integers n. b) If T is a tree with n vertices then prove that T contains no cycles, and has $n-1$ edges.	10
Q4	a) Show that the implication $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology by using truth table. b) Show that the propositions $\sim \forall x(p(x) \to q(x))$ and $\exists x(p(x) \wedge \sim q(x))$ are logically equivalent.	10
Q5	a) Give a formula for the coefficients of x^k, k is an integer, in the expansion of $(x^2-\frac{1}{x})^{100}$. b) Draw the graph whose adjacency matrix is given by $\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$	10

UNIVERSITY OF THE PUNJAB

Third Semester 2017
Examination: B.S. 4 Years Programme

Roll No. 015574

PAPER: Discrete Mathematics (IT)
Course Code: MATH-231/IT-21404

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q2. Short Questions (2x10=20)

i) Prove that an undirected graph has even number of vertices of odd degree.

ii) Show that $p \to q \equiv \neg p \vee q$.

iii) Determine whether $(\neg p \wedge (p \to q)) \to \neg q$ is a tautology.

iv) Determine whether the function $f(x) = x^2$ from $N \to N$ is one-to-one. Is this function onto?

v) Let $f: R \to R$ be defined by $f(x) = \frac{2x-7}{4}$. Find $f^{-1}(x)$.

vi) How many non-isomorphic unrooted trees are there with three vertices?

vii) Give an example of a relation which is symmetric and anti-symmetric.

viii) Show that $\neg(p \vee q)$ *and* $\neg p \wedge \neg q$ are logically equivalent.

ix) Let $P(x)$ denote the statement "$x > 3$". What are the truth values of $P(4)$ and $P(2)$?

x) Construct the truth table for $(p \wedge q) \to (p \vee q)$.

Subjective Questions (30)

Q3. a) State and prove the pigeon hole principal.

b) Show that $\neg(p \leftrightarrow q)$ *and* $p \leftrightarrow \neg q$ are logically equivalent.

Q4. a) If T is a tree with n vertices then prove that T contains no cycles and has $n-1$ edges.

b) An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

Q5. a) Draw a graph whose adjacency matrix given by

	a	b	c	d
a	0	0	1	1
b	0	0	1	0
c	1	1	0	1
d	1	1	1	0

b) Show that among any group of five integers, there are two with the same remainder when divided by 4.

Proof 4(b): Let G be a tree, then it is connected and contains no cycle ⇒ path exist between every two vertices u and v. Let P_1, P_2 be two paths between u and v.
Case I: if $P_1 \cap P_2 = \phi$ ⇒ ∃ a cycle in G (✗).
Case II: if $P_1 \cap P_2 \neq \phi$ ⇒ some edges are common in P_1 and P_2. P_1 and P_2 must diverge at some vertex (say w) and again meet at some vertex (say x) making a cycle ✗.

4(b) Conversely: If G is graph which ... between every two vertices ⇒ G is connected, G does not contain a cycle, as ... cycle ... both between ...

Def: T is tree if (i) T is connected and (ii) it contains no cycle.
Def: A graph is connected if ∃ at least one path between every two vertices.
Proof (4(a)): Result is true for T_1, T_2, T_3. Let result is true for $T_{k'}$, $T_{k''}$. Consider T_k and remove one edge from it, then $T_k = T_{k_1} \cup T_{k_2}$, where $k_1 + k_2 = k$. $|E(T_{k_1})| = k_1 - 1$, $|E(T_{k_2})| = k_2 - 1$. Now $|E(T_k)| = |E(T_{k_1})| + |E(T_{k_2})|$

UNIVERSITY OF THE PUNJAB

Roll No. ....

Third Semester 2018

Examination: B.S. 4 Years Programme

PAPER: Discrete Mathematics (IT) **TIME ALLOWED: 30 mins.**

Course Code: MATH-231/IT-21404 **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

OBJECTIVE TYPE

Q1. Encircle the correct answer (1x10=10)

1. The inverse of the conditional statement $p \to q$ is
   a. $\neg p \to q$ b. $\neg p \to \neg q$ c. $q \to p$ d. $\neg q \to \neg p$
2. If A= {1,2,3,4}, then the number of elements in P (A) =.....
   a. 2^4 b. 2^5 c. 2^6 d. 2^7
3. Consider the relation R={(1,1),(1,2),(1,4),(2,1),(2,2),(3,3)} on set A={1,2,3,4} is
   a. Symmetric b. Reflexive c. Transitive d. None of these
4. A graph of a function *f* is one-to-one if and only if every horizontal line intersects the graph in .... point.
   a. at most one b. exactly one c. at least one d. none of these
5. 5, 9, 13, 17, …is
   a. Arithmetic series b. Geometric series c. Arithmetic sequence d. Geometric sequence
6. The total number of one-to-one functions, from a set with three elements to a set with four elements is………..
   a. 24 b. 16 c. 12 d. 9
7. If $f(x) = 2x+1$ then its inverse =…………
   a. $x-1$ b. $\frac{x-1}{2}$ c. $1+x$ d. None of these
8. The inverse of given relation R = {(1,1),(1,2),(1,4),(3,4),(4,1)} is
   a. {(1,1),(2,1),(4,1),(2,3)}
   b. {(1,1),(1,2),(4,1),(4,3),(1,4)}
   c. {(1,1),(2,1),(4,1),(4,3),(1,4)}
   d. None of these
9. If a graph has vertices of degrees 1, 1, 4, 4 and 6. How many edges does the graph have?
   a. 8 b. 10 c. 12 d. 14
10. Which term of the sequence 4,1,-2,... is -77
    a. 26 b. 27 c. 28 d. None of these

UNIVERSITY OF THE PUNJAB

Third Semester 2018
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Discrete Mathematics (IT)
Course Code: MATH-231/IT-21404

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SUBJECTIVE TYPE

Q2. Solve the following short questions (2x10=20)

1. Draw two 3-regular graphs with six vertices.
2. Construct a truth table for the statement form
   $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$.
3. What is a compound statement?
4. Let X is a non-empty set. Prove that the identity function on X is bijective.
5. How many integers from 1 through 1000 are multiples of 3 or multiples of 5?
6. Find the sum of all two digit positive integers which are neither divisible by 5 nor by 2.
7. Define a binary relation P from R to R as follows:
   for all real numbers x and y $(x, y) \in P \Leftrightarrow x = y^2$. Is P a function? Explain.
8. Find x and y given $(2x, x + y) = (6, 2)$.
9. Suppose that f is defined recursively by $f(0) = 3$, $f(n + 1) = 2f(n) + 3$. Find $f(2)$.
10. Find the number m of ways that nine toys can be divided among four children if the youngest child is to receive three toys and each of the others two toys.

Q3. Solve the following **Long Questions** (5x6=30)

1. Define a relation R on the set of all integers Z as follows:
   for all integers m and n, $m \, R \, n \Leftrightarrow m \equiv n \pmod 3$.
   Prove that R is an equivalence relation.
2. Given any two distinct rational numbers r and s with $r < s$. Prove that there is a rational number x such that $r < x < s$.
3. Prove that if n is an odd integer, then $n^3 + n$ is even.
4. Let S be the function such that $S(n)$ is the sum of the first n positive integers. Give a recursive definition of $S(n)$.
5. There are 15 girls and 25 boys in a class. How many students are there in total?
6. For the complete graph K_n, find
   (i) the degree of each vertex
   (ii)the total degrees
   (iii)the number of edges.