UNIVERSITY OF THE PUNJAB

Second Semester 2011

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II
Course Code: IT-12392

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 60

Attempt this Paper on Separate Answer Sheet provided.

PART II: SHORT QUESTIONS **(30)**

Q.2.

(i) Find the equation of the sphere having points $(1, 2, 3)$ and $(1, -2, 0)$ as one of its diameter end points. ?

(ii) Find the parametric equations of the line passing through the point $(-1, 0, 3)$ and parallel to the vector $\hat{i} + 2\hat{j} + \hat{k}$.

(iii) Determine whether the line $x = -1 + 2t,\ y = 4 + t,\ z = 1 - t$ and plane $4x + 2y - 2z = 8$ are parallel, perpendicular, or neither.

(iv) Evaluate the definite integral $\int_0^2 \|t\hat{i} + t^2\hat{j}\|$.

(v) $f(x, y) = y^{-3/2}\tan^{-1}(\frac{x}{y})$. Find $f_y(x, y)$. $4x^3y^2+2y-z=0$

(vi) Find an equation for the tangent plane to the surface $z = 4x^3y^2 + 2y$ at the point $P(-1, -2, 10)$.

(vii) Evaluate the double integral $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\,dA$ over the rectangular region $R = \{(x, y) : 0 \le x \le 1,\ 2 \le y \le 3\}$. /2

(viii) $\vec{F} = x^2\hat{i} - 2\hat{j} + yz\hat{k}$. Find curl $\vec{F}$.

(ix) State Green's Theorem.

(x) State Stokes' Theorem.

PART III: SUBJECTIVE QUESTIONS **(30)**

Q.3.

(i) Find the distance between the given skew lines $x = 1 + 7t,\ y = 3 + t,\ z = 5 - 3t,\ x = 4 - t,\ y = 6,\ z = 7 + 2t$. 7/59

(ii) Find the unit tangent, unit normal and unit binormal to the curve $\vec{r}(t) = (\sin t - t\cos t)\hat{i} + (\cos t + t\sin t)\hat{j} + \hat{k})$.

(iii) A manufacturer makes two models of an item, standard and deluxe. It costs \$40 to manufacture the standard model and \$60 for the deluxe. A market research firm estimates that if the standard model is priced at x dollars and the deluxe at y dollars, then the manufacturer will sell $500(y - x)$ of the standard items and $45,000 + 500(x - 2y)$ of the deluxe each year. How should the items be priced to maximize the profit?

(iv) Find the area of the portion of the paraboloid $z = 1 - x^2 - y^2$ that is above the xy-plane.

(v) Find the work done by the force field $\vec{F}(x, y) = (x^2 + xy)\hat{i} + (y - x^2y)\hat{j}$ on a particle that moves along the curve $C:\ x = t, y = \frac{1}{t},\ 1 \le t \le 3$.

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MAX. MARKS: 60

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Part-II: Short Questions:-
Note: Each Question carries 5 Marks.

Q.2. Find parametric equations of the line tangent to the graph of $\vec{r}(t) = e^{2t}\hat{i} - 2\cos 3t\hat{j}$ at the point where $t = 0$.

Q.3. Find the arc length of the parametric curve $x = e^t$, $y = e^{-t}$, $z = \sqrt{2}t$; $0 \le t \le 1$.

Q.4. The volume of a right circular cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Show that if the height remains constant while the radius changes, then the volume satisfies $\frac{\partial V}{\partial r} = \frac{2V}{r}$.

Q.5. Let $T = x^2y - xy^3 + 2$; $x = r\cos\theta$, $y = r\sin\theta$. Use appropriate form of the chain rule to find $\frac{\partial T}{\partial r}$.

Q.6. Evaluate $\displaystyle\int_0^3 \int_0^{\sqrt{9-z^2}} \int_0^x xy\, dy\, dx\, dz.$

Q.7. Evaluate the line integral $\displaystyle\int_C \frac{1}{1+x}\, ds$ along the parametric curve C :, $x = t$, $y = \frac{2}{3}t^{3/2}$ $(0 \le t \le 3)$

Part-III: Subjective Questions:-
Note: Each Question carries 10 Marks.

Q.8. Find the equation of the plane that contains the line $x = 3t$, $y = 1 + t$, $z = 2t$ and is parallel to the intersection of the planes $2x - y + z = 0$ and $y + z + 1 = 0$.

Q.9. Find all points on the portion of the plane $x + y + z = 5$ in the first octant at which $f(x, y, z) = xy^2z^2$ has a maximum value.

Q.10. Find the surface area of the portion of the paraboloid $\vec{r}(u, v) = u\cos v\hat{i} + u\sin v\hat{j} + u^2\hat{k}$ for which $1 \le u \le 2$, $0 \le v \le 2\pi$.

UNIVERSITY OF THE PUNJAB

Second Semester 2013

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II
Course Code: IT-12392 / MATH-132

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Attempt this paper on separate answer sheet provided.

Q.2. **Do the following "Short Answer":**

(a) Find the point of intersection of the lines $x = t,\ y = -t+2,\ z = 1+t$ and

$$x = 2s+2,\ y = s+3,\ z = 5s+6.$$

(b) Show that the curvature of a circle of radius a is $\dfrac{1}{a}$.

(c) Find the distance from the plane $x+2y+6z=1$ to the plane $x+2y+6z=10$.

(d) What is the domain and range of a function $w=\sqrt{y-x^2}$?

(e) Find the derivative of $f(x,y,z)=xy+yz+zx$ at $p_0(1,-1,2)$ in the direction of $\overline{A}=3\hat{i}+6\hat{j}-2\hat{k}$.

Q. 3 Do the following "Long Question".

I. Evaluate $\int_1^e\int_1^e\int_1^e \frac{1}{xyz}\,dxdydz$ and $\int_1^e\int_1^e\int_1^e \ln r \ln s \ln t\, dt\, dr\, ds.$ $(rst-space)$

II. Find a plane through $p_0(2,1,-1)$ and perpendicular to the line of intersection of the planes,
$2x+y-z=3,\quad x+2y+z=2.$

III. Write Stokes' Theorem and verify it for the hemisphere $S: x^2+y^2+z^2=9,\ z\geq 0$, its bounding circle $C: x^2+y^2=9,\ z=0$, and the field $\overline{F}=y\hat{i}-x\hat{j}$.

UNIVERSITY OF THE PUNJAB

Second Semester 2014

Examination: B.S. 4 Years Programme

Roll No. [redacted]

PAPER: Calculus (IT)-II
Course Code: MATH-132 / IT-12392

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

Short Questions [20]

Q.02

(a) Find equation of sphere having points $(1,2,3)$ and $(1,-2,0)$ as one of its diameter.

(b) Evaluate $\int_3^4 \int_1^2 \frac{1}{(x+y)^2}\,dy\,dx$

(c) State Stokes Theorem and define curvature function of the curve.

(d) State divergence theorem and find $f_y(x,y)$ if $f(x,y) = e^{-x}\cos\left(\frac{3}{y}\right)$.

(e) Find the vector projection of $\underline{u} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ onto $\underline{v} = \hat{i} - 2\hat{j} - 2\hat{k}$ and the scalar component of $\underline{u}$ in the direction of $\underline{v}$.

Long Questions

Q.03 (a) Find the distance from the plane $x + 2y + 6z = 1$ to the plane $x + 2y + 6z = 10$. [10]

(b) Show that the curvature of a circle of radius a is $\frac{1}{a}$.

Q.04 Derive the equation for plane in vector form and find an equation for the plane passing through the point $P_0(2,4,5)$ and perpendicular to the line $x = 5 + t,\ \ y = 1 + 3t,\ \ z = 4t$. [10]

Q.05 Evaluate $\oint_C x\cos y\,dx - y\sin x\,dy$ using Green's theorem where C is the square with vertices $(0,0)$, $(\frac{\pi}{2},0)$, $(\frac{\pi}{2},\frac{\pi}{2})$, $(0,\frac{\pi}{2})$ [10]

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UNIVERSITY OF THE PUNJAB

Roll No. ....

Second Semester 2015
Examination: B.S. 4 Years Programme

PAPER: Calculus (IT)-II | **TIME ALLOWED: 30 mins.**
Course Code: MATH-132 / | **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Objective Type

Q. 01 **Chose the best answer.** **[10]**

i) The distance of the point $(3,1,-2)$ to the plane $4x-3y+5=0$ is -----------

a) 13 b) $\frac{14}{5}$ c) 26 d) $\sqrt{14}$

ii) The acute angle between the planes $2x+y-z-5=0$ and $x-y-2z+5=0$ is -----

a) 30° b) 45° c) 60° d) 75°

iii) If x,y, and z are independent variables and $f(x,y,z)=x\sin(y+3z)$, then f_y is

a) $-3x\cos(y+3z)$ b) $3x\cos(y+3z)$ c) $3\cos(y+3z)$ d) $x\cos(y+3z)$

iv) The gradient vector of $f(x,y)=x^2\sin 2y$ at the point $(1, \pi/2)$ is

a) $0\hat{i}+2\hat{j}$ b) $0\hat{i}-2\hat{j}$ c) $\hat{i}+2\hat{j}$ d) $-\hat{i}+2\hat{j}$

v) A spherical coordinate equation for the cone $z=\sqrt{x^2+y^2}$ is

a) $\phi=\frac{\pi}{2}$ b) $\phi=\frac{\pi}{3}$ c) $\phi=\frac{\pi}{4}$ d) $\phi=\frac{\pi}{6}$

vi) A surface presented by the equation $\rho=4$ is a

a) cone b) sphere c) line d) plane

vii) The curvature of a straight line is ....

a) zero b) one c) two d) infinite

viii) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$ represents a surface of the type

a) a paraboloid b) an elliptic cone c) a hyperboloid d) a cylinder

ix) The domain of function $w=xy\ln z$ is

a) $(x,y,z)\neq(0,0,0)$ b) Entire Space c) half space $z>0$ d) half space $z<0$

x) The equation $x^2+y^2-z^2=16$ in cylindrical coordinates is

a) $r^2=4$ b) $r^2+z^2=16$ c) $r^2-z^2=4$ d) $r^2-z^2=16$

UNIVERSITY OF THE PUNJAB

Second Semester 2015
Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Calculus (IT)-II
Course Code: MATH-132

TIME ALLOWED: 2 hrs. & 30 mins.
MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

	Short Questions	
Q.02	**(a)** Find the center and radius of the sphere $x^2+y^2+z^2+4x-4z=0$. **(b)** Evaluate $\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2}} dz\,dy\,dx$ **(c)** State Stokes Theorem and define curvature function of the curve. **(d)** State divergence theorem and find $\frac{\partial z}{\partial x}$ if the equation $yz-\ln z=x+y$ defines z as a function of the two independent variables x and y and the partial derivative exists. **(e)** Find the vector projection of $\underline{u}=5\hat{j}-3\hat{k}$ onto $\underline{v}=\hat{i}+\hat{j}+\hat{k}$ and the scalar component of $\underline{u}$ in the direction of $\underline{v}$.	[20]
	Long Questions	
Q.03	**(a)** Find the distance from the point $(2,-3,4)$ to the plane $x+2y+2z=13$. **(b)** Show that the curvature of a circle of radius a is $\frac{1}{a}$.	[10]
Q.04	Derive the equation for plane in vector form and find an equation of the plane passing through $(5,-1,4)$ and perpendicular to each of the planes $x+y-2z-3=0$ and $2x-3y+z=0$.	[10]
Q.05	Evaluate $\int_C (x-y)dx+(x+y)dy$ counterclockwise around the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$.	[10]

UNIVERSITY OF THE PUNJAB

Second Semester 2016

Examination: B.S. 4 Years Programme

Roll No. 15570

TIME ALLOWED: 2 hrs. & 30 mins.

Calculus (IT)-II

Code: MATH-132 / IT-12392

MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

	Short Questions	[20]
Q.02	(a) Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P_0(1,1,0)$ in the direction of $\underline{v} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. (b) Define tangent plane and normal line. (c) Show that the curvature of a circle of radius a is $\frac{1}{a}$. (d) State divergence theorem and find $\frac{\partial x}{\partial z}$ if the equation $yz - \ln z = x + y$ defines x as a function of the two independent variables y and z and the partial derivative exists. (e) Find the vector projection of $\underline{u} = 3\hat{j} + 4\hat{k}$ onto $\underline{v} = 10\hat{i} + 11\hat{j} - 2\hat{k}$ and the scalar component of $\underline{u}$ in the direction of $\underline{v}$. $\text{proj} = \frac{U \cdot V}{\|V\|}$	
	Long Questions $\overline{PQ}$	
Q.03	(a) Find the parametric equations for the line through $P(-3, 2, -3)$ and $Q(1, -1, 4)$. (b) Find the unit tangent vector of the curve $\vec{r}(t) = t^2\hat{i} + (2\cos t)\hat{j} + (2\sin t)\hat{k}$.	[10]
Q.04	(a) Find an equation of the plane passing through $(5, -1, 4)$ and perpendicular to each of the planes $x + y - 2z - 3 = 0$ and $2x - 3y + z = 0$. (b) Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^{8-x^2-y^2} dz\, dy\, dx$ $\frac{r'(t)}{\|r'(t)\|}$	[10]
Q.05	Find the greatest and smallest values that the function $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.	[10]

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

UNIVERSITY OF THE PUNJAB

Second Semester - 2017

Examination: B.S. 4 Years Programme

Roll No. [illegible]

TIME ALLOWED: 2 hrs. & 30 mins.

MAX. MARKS: 50

Calculus (IT)-II

Code: MATH-132 / IT-12392

Attempt this Paper on Separate Answer Sheet provided.

[20]

Short Questions

PQ

Q 2. (a) Find parametric equation of line passing through $P(1,0,0)$ and $Q(3,2,1)$.

(b) Show that curvature of a line is always 0.

(c) Find the arc length of the curve $r(t) = < t, \sqrt{t+2} >$ from $1 \le t \le 7$. 12/3

(d) Determine whether the vector field $f = < 1, y, x >$ is conservative?

(e) Show that divergence of a curl of a vector field is always zero. $x+y+z$

$\| \partial/\partial x + \partial/\partial y + \partial/\partial z \|$

x
y
z

[30]

Long Questions

Q 3. (a) Find unit tangent and unit normal vector for the curve $r(t) = < \cos(t), \sin(t) >$ at $t = \frac{\pi}{2}$.

(b) Find potential function for the vector field $f = < x, y, z >$.

Q 4. (a) Calculate the flow through the closed surface using Divergence theorem.

derivative

$$f(x,y,z) = < x+y, y, z >: z = 16 - x^2 - y^2, \qquad z = 0$$

$\frac{u \cdot v}{|v|}$

(b) Find the projection of $u = < 1, -1, 3 >$ on $v = < 3, 3, 0 >$.

Q 5. Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ for $w = x + y^3 - 2z^2 + 2xy - 5xz - 4yz$, where $x = 2s - 3t$, $y = 4s + t$, $z = s + 5t$.

Write your answers in terms of t.