UNIVERSITY OF THE PUNJAB

B.S. 4 Years Program : First Semester – Fall 2021

Roll No. .....

Subject: Pre-Calculus – I **Paper: MD-001** **Time: 3 Hrs.** **Marks: 60**

Q.1. Solve the following questions. (15x2=30)

1. (2 points) Let $f(x) = \frac{\sqrt{x^2-16}}{x+12}, x \neq -12$. Find domain and range of f.
2. (2 points) Given $f(x) = x^3 - ax^2 + bx + 1$. If $f(2) = -3$ and $f(-1) = 0$. Find the value of a and b.
3. (2 points) Find inverse function of $f(x) = 2 + \sqrt{x-1}$.
4. (2 points) Evaluate $\lim_{x\to 0} \frac{\sin 7x}{x}$.
5. (2 points) Draw graph of $y = \begin{cases} x+3 & \text{if} \quad x \neq 3 \\ 2 & \text{if} \quad x = 3 \end{cases}$
6. (2 points) If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, show that $A^4 = I_2$.
7. (2 points) Show that $\begin{vmatrix} x & a+x & b+c \\ x & b+x & c+a \\ x & c+x & a+b \end{vmatrix} = 0$.
8. (2 points) Show that, $A = \begin{bmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{bmatrix}$ is a singular matrix.
9. (2 points) What is the circular measure of the angle between the hands of a watch at 4 O′clock?
10. (2 points) Prove that $\cot^4\theta + \cot^2\theta = \csc^4\theta - \csc^2\theta$, where θ is not an integral multiple of $\frac{\pi}{2}$.
11. (2 points) Find the value of $\cos\frac{\pi}{12}$.
12. (2 points) Prove that $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.
13. (2 points) Find the period of $3\cos\frac{x}{5}$.
14. (2 points) Find the center and radius of the circle having equation,

$$4x^2 + 4y^2 - 8x + 12y - 25 = 0.$$

15. (2 points) Find the center, foci, eccentricity, vertices and equations of the directrices of the ellipse having equation, $9x^2 + y^2 = 18$.

Q.2. Solve the following questions. **(5x6=30)**

1. Prove that, $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e$.

2. Show that $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = (x+3)(x-1)^2$.

3. Find rank of the matrix, $A = \begin{bmatrix} 3 & 2 & -9 & 5 \\ 4 & 8 & 9 & -1 \\ 3 & -1 & 0 & -1 \\ 0 & 3 & 0 & 0 \end{bmatrix}$

4. Prove that $(\tan\theta + \cot\theta)^2 = \sec^2\theta \csc^2\theta$.

5. Prove that $\frac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi} = \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$.